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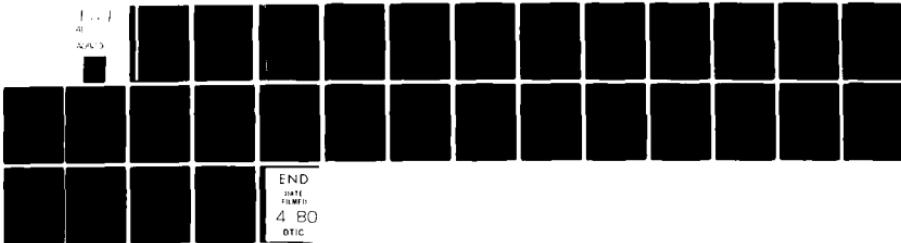
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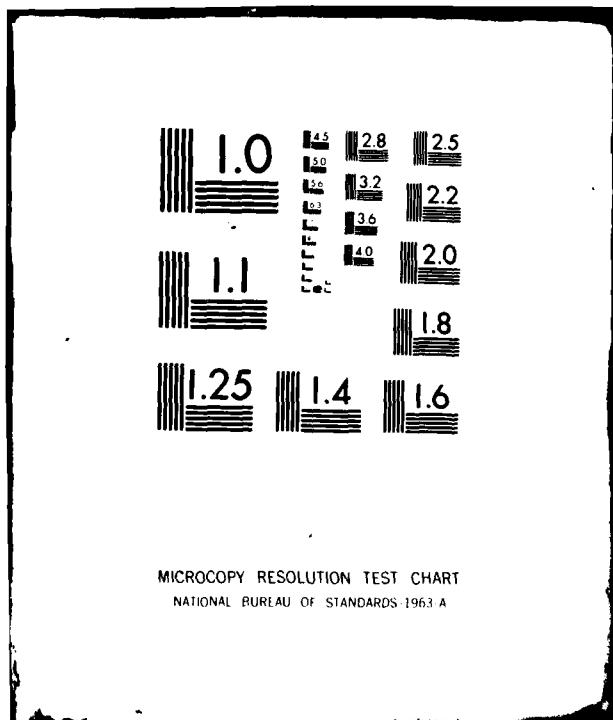
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TWO MODELS DESCRIBING THE DETECTION PERFORMANCE  
OF A DISTRIBUTED FIELD OF PASSIVE SENSORS

by

NIELS BACHE and BERT van DOMSELAAR

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OF A DISTRIBUTED FIELD OF PASSIVE SENSORS.

by

10 Niels Bache and Bert van Domselaar

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TWO MODELS DESCRIBING THE DETECTION PERFORMANCE  
OF A DISTRIBUTED FIELD OF PASSIVE SENSORS

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Niels Bache and Bert van Domselaar

ABSTRACT

A simulation model and an analytical model for studying the detection performance of a distributed field of passive sensors against conventional and nuclear submarines are presented. The detection performance is measured by the mean detection probability and the standard deviation of the detection probability. A mutual check of both models has been performed and results of the comparisons are discussed.

INTRODUCTION

Following on from a previous study on the defence of the Southwestern Approaches against submarines entering CINCHAN's area of command [1], a new study on the subject of distributed fields of passive sensors has been initiated. Because of the high noise background in areas like the English Channel, which affects virtually all passive sensor systems, the detection range for each sensor is bound to be short. This could be considered as an operational advantage, because the resulting good datums greatly help the localization and attack phases.

Based on this idea, the aim of this study has been formulated as investigating the detection and localization performance and effectiveness of a distributed field of passive, long-life sensors versus nuclear and conventional submarines, patrolling or transiting in choke-points such as the English Channel. The study has advanced to an analytical and a simulation model, both aimed at obtaining the detection performance of a field of passive sensors. The simulation model will be extended to submodels for localization and attack phases. Only the submodel of the simulation model representing the detection phase is discussed in this memorandum. The analytical model covers only the detection phase, since it is considered difficult, analytically, to describe all possibilities in the localization and attack phases. An analytical and a simulation model together increase the understanding of the problem more than any of the single models alone. Furthermore, a mutual check can be performed and thereby increase the confidence in the models.

This memorandum should be considered as an interim document, discussing the models for the detection performance of a distributed field of passive sensors and presenting a comparison between the models.

1 SIMULATION MODEL1.1 General

The frame of the comprehensive simulation model covering the three phases is given in Fig. 1. Obviously the three phases can be treated individually and separate models can be developed, taking into account that the models have to be linked via their input and output as shown in the figure. The structure and the optional choices of the detection phase model are discussed in the next chapter.

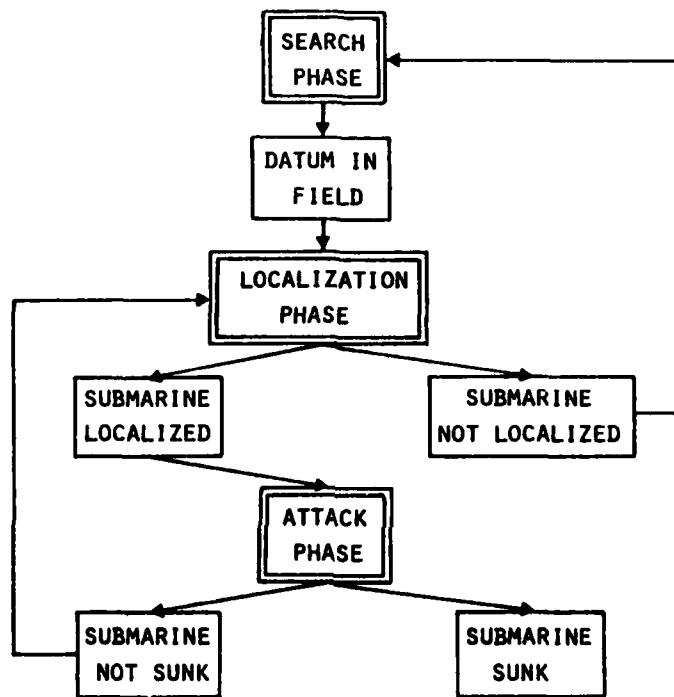


FIG. 1 PHASES IN SIMULATION MODEL

1.2 Submodels of Search-Phase Model

The following requirements for the model have been derived from the objectives of the study. The model should be able to handle:

- Different types of passive sensors, which can be distributed in any type of pattern, randomly or orderly.
- Different types of searchers (e.g. aircraft, ships), which can travel in an area both stochastically and deterministically.
- Conventional and nuclear submarines, which can travel in an area both stochastically and deterministically.

- Different snorkelling policies for a conventional submarine, taking into account the charge and discharge characteristics of its batteries.
- Any type of monitoring schedule for the sensor field, i.e. if it is not possible to monitor all sensors simultaneously, one should be able to choose a monitoring schedule according to a time-sharing system.
- False targets having similar noise characteristics to those of a submarine.
- Any type of detection curve for the passive sensors against nuclear and conventional submarines and false targets under different environmental conditions. This requirement, for instance, makes it possible to deal with the shallow-water environment and different shipping densities in the English Channel, which will have main influence on the background noise of the passive sensors and therefore also on the detection curves.

The requirements described above already indicate some of the submodels. Figure 2 shows the flowchart of the search-phase. Each box, representing a submodel, is discussed individually.

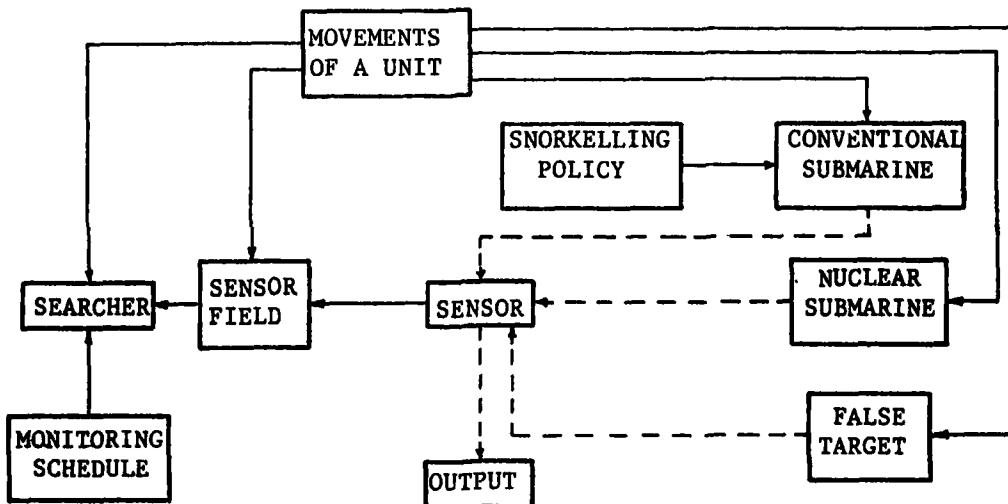


FIG. 2 THE SEARCH-PHASE MODEL, illustrating how the component sub-models are linked

### 1.2.1 'Movements of a Unit' Submodel

In a simulation run, copies of this submodel are applied to each moving unit, which can be submarines, false targets, searchers, or sensor fields. The movement of a unit can either be predetermined or random, but in both cases the movements are restricted to straight lines and arcs. Moreover its movement is interrupted when the edge of its predetermined travel area is crossed; it is then bounced back into the area in a random direction.

Figure 3 shows an example of a track of a unit in an area.

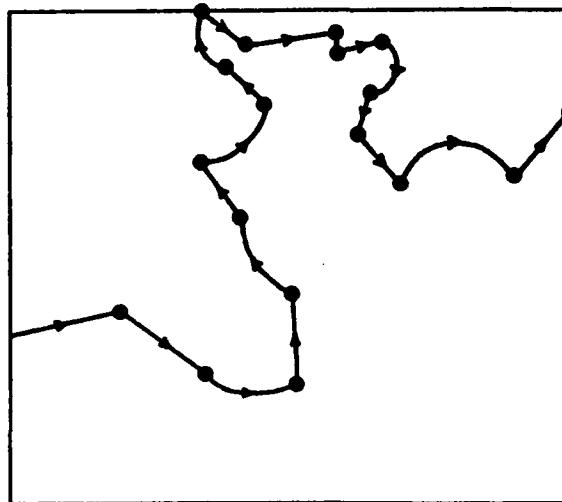


FIG. 3 EXAMPLE OF A TRACK OF A UNIT IN AN AREA

#### 1.2.2 'Nuclear Submarine' Submodel

A "movements of a unit" submodel (see Sect. 1.2.1) establishes the movements of a nuclear submarine. The 'nuclear submarine' submodel carries information about the noise characteristics of the submarine and is used to calculate the detection probability when the submarine is passing a sensor.

#### 1.2.3 'Conventional Submarine' Submodel

The movements for this submodel are established in the same way as for a nuclear submarine. Only the noise characteristics for the snorkelling state are needed, because the conventional submarine is assumed to be silent while submerged. For this, a copy of the "snorkelling policy" submodel (Sect. 1.2.4) is applied. Since the speed possibilities of the submarine in the two states (snorkelling and submerged) are completely different, an extra speed parameter has been introduced such that the submerged speed and snorkelling speed can individually be chosen randomly or deterministically.

#### 1.2.4 'Snorkelling Policy' Submodel

In this submodel, of which a copy is attached to the "conventional submarine" submodel (Sect. 1.2.3), a decision is made as to whether the submerged state of the submarine is changed from snorkelling to silent or viceversa. Two essentially different policies have been implemented:

##### a. The minmax policy

This policy is based on imposing upper and lower limits on the battery capacity, which cannot be exceeded. When the submarine is snorkelling and the upper limit is reached, the submarine dives; when the submarine is submerged and the lower limit is reached, the submarine starts snorkelling. To apply this policy, the charge and discharge characteristics of the

battery must be known. Charging and discharging mainly depend on the speed of the submarine: non-linearly for charging and linearly for discharging. Curves (taken from [2]) are used that show these relations. They also allow the choice of some parameters according to the type of submarine.

### b. The negative exponential policy

Due to an air threat for conventional submarines, the lengths of the snorkelling and submerged periods are regarded as being negative exponentially distributed [3]. The snorkelling periods  $t_s$  are distributed according to the density function

$$f(t_s) = \frac{1}{T_s} e^{-\frac{t_s}{T_s}},$$

where  $T_s$  is the mean snorkelling time.

Similarly, for the submerged periods the density function is

$$g(t_d) = \frac{1}{T_d} e^{-\frac{t_d}{T_d}},$$

where  $T_d$  is the mean submerged time.

Draws are taken from the two distributions in turn in order that submerged and snorkelling periods will alternate. With this policy the lengths of the snorkelling and the submerged period are two random variables in the simulation.

Besides one of the two described policies the submarine can also use a policy that is a mixture of the two. The lengths of the snorkelling and submerged periods are negative exponentially distributed, but this process is interrupted and the state of the submarine is changed when the upper or lower limit of the battery is exceeded. This mixture policy is supposed to be the most realistic one.

#### 1.2.5 'False Target' Submodel

The movements of a false target are established by means of a copy of the "movements of a unit" submodel (Sect. 1.2.1). Since only false targets with similar noise characteristics to those of a submarine are considered, it is expected that the information needed to calculate the detection curves will be similar to that of a nuclear or conventional submarine.

#### 1.2.6 'Searcher' Submodel

In the simulation each searching unit is monitoring one or more sensors. Therefore one copy of the "sensor field" submodel (Sect. 1.2.7) is attached to each searcher. Moreover, the "searcher" submodel gets a copy of the "monitoring schedule" submodel (Sect. 1.2.8) when the field is too big to monitor all sensors simultaneously; a time-sharing schedule is then applied.

Although a copy of the "movements of a unit" submodel (Sect. 1.2.1) is attached to the "searcher" submodel to permit the searcher to move randomly or deterministically, an option allows the searcher to have a fixed position during the simulation. This could be applicable for a field with cabled sensors, monitored from an on-shore station. (Note that the on-shore station is the searcher).

Different copies of the "searcher" submodel can be created to allow more than one searcher to be deployed. For instance, an area can be subdivided so that each searcher monitors a sensor field in a sub-area.

#### 1.2.7 'Sensor Field' Submodel

Two optional fields have been implemented in the "sensor field" submodel. The first is a field in which the sensors have fixed positions or move independently of the movements of a searcher, as could be caused by drifting. A copy of the "movements of a unit" submodel (Sect. 1.2.1) is then attached to the "sensor field" submodel. With the second option the field has the same movements as its "searcher" during the whole simulation, as would occur when a ship is towing an array.

In this submodel the individual sensors of the field are created by attaching copies of the "sensor" submodel (Sect. 1.2.9), each of which needs such information as its initial position and detection characteristics.

#### 1.2.8 'Monitoring Schedule' Submodel

If the monitoring capabilities of the searcher are not big enough to monitor all sensors in a field simultaneously, a time-sharing system has to be applied. Input parameters for this submodel are the number of sensors monitored simultaneously and the period for which a set of sensors is monitored.

Both deterministic and random monitoring schedules have been implemented. The deterministic schedule requires that the order of the sensors monitored in each successive period is given until a cycle has been established; this cycle is then repeated throughout the simulation. With a random schedule, the sensors to be monitored in each monitoring period are chosen randomly, independently of the previous period. This implies that one sensor can be monitored for more than one period successively.

#### 1.2.9 'Sensor' Submodel

The detection of a submarine or false target occurs in this submodel. In the simulation a "sensor" will be active only when it is monitored and when a target, i.e. a submarine or false target, is inside the field. The detection probability is calculated from the noise characteristics of the target and the system characteristics of the sensor. An important system input parameter of this submodel is the minimum time necessary to monitor a target before a detection opportunity is considered to exist.

This calculated detection probability can be used in two ways. The first is to accumulate the probability values during one simulation run. This

gives the detection probability of the field for that particular run. For instance, if a sensor field is monitored in which submarines are entering with a certain arrival rate, the detection rate of the field could be calculated.

The second way of using the calculated detection probability of a sensor is by drawing a random number uniformly distributed in the interval  $[0,1]$ . If the sample value is greater than the detection probability, the target is considered undetected; if the sample value is smaller than the detection probability, the target is considered detected. When models of the localization and attack phases exist, the localization phase model should be entered after a detection.

### 1.3 Programming

The computer program for the simulation model has been written in the SIMULA language. A nice feature of this language is its time axis, on which future events can be scheduled. Each submodel can work individually and at the end of its activity it can be decided when it will be active again. This event is scheduled on the SIMULA time axis and the submodel remains dormant until that moment. Moreover, a dormant submodel can always be activated in a later event.

## 2 THE ANALYTICAL MODEL

### 2.1 General

The analytical model describes only the detection phase for a given field of passive sensors, i.e. for a number of passive sensors distributed in a given fashion in an area.

The detection probability,  $P$ , of the field depends first of all on two groups of factors. The first group is the field, where the elementary unit is the sensor; the second is the sensor itself. This division is maintained throughout the description of the analytical model.

In general, without any assumptions,  $P$  can be written

$$P = 1 - [1 - \text{Prob}\{D_1\}] \cdot [1 - \text{Prob}\{D_2 | \bar{D}_1\}] \cdot [1 - \text{Prob}\{D_3 | \bar{D}_1 \cap \bar{D}_2\}] \cdot \dots \cdot [1 - \text{Prob}\{D_x | \bar{D}_1 \cap \bar{D}_2 \cap \dots \cap \bar{D}_{x-1}\}],$$

where  $D_i$  is the event that sensor number  $i$  detected the submarine,

$i$  is a sensor counter starting from the submarine entrance to the field for those sensors where the submarine comes within range  $R$  of the sensor,

$\bar{D}_i$  is the negation of  $D_i$ ,

$|$  and  $\cap$  mean, respectively, "given" and "and",

$x$  is the number of sensors encountered by the submarine (in the way described above) during its stay inside the field.

Without any explicit statement about the dependence of the event  $D_i$  on the event  $\bar{D}_1 \cap \bar{D}_2 \cap \dots \cap \bar{D}_{i-1}$ , only limiting cases can be considered. These are all related to the independence case:

$$\text{Prob}\{D_i | \bar{D}_1 \cap \bar{D}_2 \cap \dots \cap \bar{D}_{i-1}\} = \text{Prob}\{D_i\}.$$

Therefore only the independence case is considered in the analytical model, with  $P$  written as

$$P = 1 - (1-p_1)(1-p_2) \dots (1-p_x),$$

[Eq. 1]

where the  $p$ 's – the detection probability for each sensor – can be considered in isolation, i.e. independent of events that happened previously.

The  $p$ 's and  $x$  will be regarded as stochastic variables. Constants can often be regarded as a special case of random variables and this will be used below.  $P$  will then be a stochastic variable, which we will express by calculating the mean  $P_M$  of  $P$  and the standard deviation  $P_{SD}$  of  $P$  (instead of considering the whole distribution for  $P$ ).

## 2.2 Field Models

When the  $p$ 's are regarded as stochastic variables they are assumed to have the same mean and variance, written respectively as  $E(p)$  and  $E(p^2) - E(p)^2$ , where  $E(\cdot)$  denotes the expected value. When the mean and variance are not the same, the formulas below can easily be extended, but at the expense of increasing algebraical complexity. Often the sensor characteristics and the circumstances under which the submarine encounters the sensors are the same for all sensors and this justifies choosing the same mean and variance.

Two field models,  $F1$  and  $F2$ , given below, are believed to cover most situations occurring in practice. The model  $F2$  is in fact a limiting case of  $F1$ , but due to the different algebraic appearance it is chosen as a separate model.

### 2.2.1 Field Model ( $F1$ )

This field model covers the following four cases:

- x constant or binomial distributed,
- p's constant or stochastic variables.

The following scenario applies (Fig. 4): the sensor field consists of  $M$  parallel lines of sensors, each line having the same fixed and known spacing "S" between sensors. The lines are displaced relative to each other in an independent random way. The submarine is assumed to cross all  $M$  lines under the same fixed and known angle " $\alpha$ ". The maximum detection range of the sensors is called  $R$ .

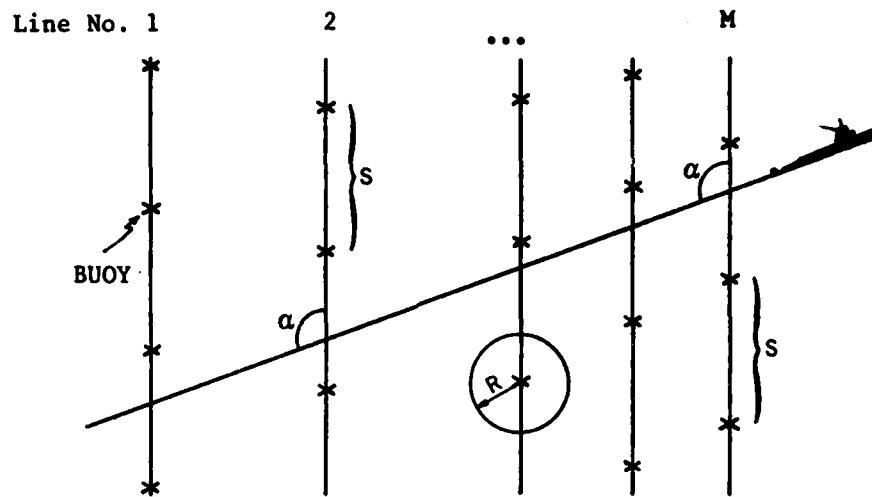


FIG. 4 PARALLEL LINES OF BUOYS

An important assumption is that the submarine must not intercept or touch more than one circle of radius  $R$  per line of sensors; otherwise there will be dependence between sensors and the model will not be directly applicable. This assumption leads to

$$2R \leq S \cdot \sin\alpha$$

and

$$m/M = 2R/(S \cdot \sin\alpha),$$

where  $m$  is the mean value of the number of sensors encountered. The distance between lines, and the number of sensors in each line, do not matter for this model. When aircraft are dropping sensor buoys in lines with sensor spacing that is accurate compared with the distance and relative position between lines, the model will be a good approximation.

If  $x$  takes the maximum value  $M$  and the mean value is  $m$ , the density function for  $x$  is then binomial,

$$\binom{M}{x} \left(\frac{m}{M}\right)^x \left(1 - \frac{m}{M}\right)^{M-x}.$$

With this density function we find

$$P_M = 1 - [1 - \frac{m}{M} \cdot E(p)]^M \quad [\text{Eq. 2a}]$$

$$P_{SD} = \sqrt{[1 - 2\frac{m}{M} \cdot E(p) + \frac{m}{M} \cdot E(p^2)]^M - [1 - \frac{m}{M} \cdot E(p)]^{2M}}. \quad [\text{Eq. 2b}]$$

Special cases are  $x$  constant, i.e.  $m = M$ , and/or the  $p$ 's constants, i.e.  $E(p^2) = E(p)^2$ . If both these conditions are fulfilled we are back to Eq. 1 and  $P_{SD} = 0$ .

### 2.2.2 Field Model (F2)

The field model covers the following two cases:

- poisson distributed
- $p$ 's constants or stochastic variables.

The following scenario applies: the sensor field with area  $A$  consists of  $N$  sensors uniformly distributed within the field.

This field model is derived from field model F1 (Sect. 2.2.1) by letting

$$M \rightarrow \infty \quad \text{and} \quad m = \frac{N}{A} \cdot 2R\ell,$$

where  $\ell$  is the total path length of the submarine inside the field.

One finds

$$P_M = 1 - e^{-\frac{N}{A} \cdot 2R\ell \cdot E(p)}$$

and

$$P_{SD} = (1-P_M) \cdot \sqrt{e^{+\frac{N}{A} \cdot 2R\ell \cdot E(p^2)}} - 1.$$

The three cases

- $x$  constant,
- binomial distributed,
- poisson distributed,

represent increasing randomness of the field.

### 2.3 Sensor Models

In this section we will again regard the  $p$ 's as stochastic variables (special case: constants), and we will find  $E(p)$  and  $E(p^2)$  in a number of cases that are believed to be representative.

From a model point of view the nuclear submarine can be regarded as a special case of the conventional submarine (infinite long snorkelling or submerged periods); therefore only the conventional submarine will be considered in the following.

The following factors contribute to the randomness of the p's:

- Snorkelling and submerged period
- Monitoring schedule
- Submarine path relative to the sensor
- Detection range of sensor

These are described in Sects. 2.3.1 to 2.3.4.

### 2.3.1 Snorkelling and Submerged Periods

Due to the likelihood of air threat, we regard the snorkelling and submerged periods of the conventional submarine as negative exponential distributed, as discussed in [3].

The mean snorkelling and submerged periods are called  $T_s$  and  $T_d$  respectively and are assumed to be strictly alternating, i.e. a snorkelling period is always followed by a submerged period and vice versa.

We assume a "cookie-cutter" detection model with radius  $R$  and probability of detection  $p_D$  for each sensor (this assumption will later be relaxed) and that opportunities for detection exist only when the submarine is snorkelling. With these two assumptions, the total snorkelling time  $t_s$  for a given time interval  $t^*$  (for example the time the submarine spends inside the cookie-cutter detection range) is of interest. The density function  $f$  for  $t_s$  for the above-described process is found to consist of two strictly alternating Poisson processes (see Appendix A):

$$f(t_s | t^*, T_s, T_d) = \frac{T_d}{T_s + T_d} \cdot g(t_s | t^*, T_s, T_d) + \frac{T_s}{T_s + T_d} \cdot g(t^* - t_s | t^*, T_d, T_s),$$

where

$$g(t_s | t^*, T_s, T_d) = e^{-\frac{t^*}{T_d}} \left[ \delta(t_s) + e^{-\left(\frac{1}{T_s} - \frac{1}{T_d}\right)t_s} \left( \frac{\theta}{2T_s} \cdot I_1(\theta) + \frac{1}{T_d} \cdot I_0(\theta) \right) \right]$$

and

$$\theta = 2 \sqrt{\frac{t_s(t^* - t_s)}{T_s T_d}}.$$

Since the snorkelling and submerged speed are in general different, and detection opportunities exist only inside range  $R$  of the sensors, we have to work in space instead of time. This is done by making the following transformation:

$$\begin{aligned} t_s &\rightarrow x_s && \text{(distance)} \\ t^* &\rightarrow y && \text{(distance)} \\ T_s &\rightarrow U_s \cdot T_s && \text{(mean snorkelling distance)} \\ T_d &\rightarrow U_d \cdot T_d && \text{(mean submerged distance),} \end{aligned}$$

where  $U_s$  and  $U_d$  are the snorkelling and submerged speed of the submarine respectively. The density function for  $\ell_s$  is then called

$$f(\ell_s | y, T_s, T_d, U_s, U_d).$$

### 2.3.2 Monitoring Schedule

Two monitoring schedules can be used as alternatives.

#### a) Random Monitoring Schedule

Operationally the next group of sensors to be monitored is chosen independently of those sensors that have or are being monitored; for example, there can be overlap between groups.

For detection opportunity we have (independence assumed)

$$P = 1 - (1 - p_D)^z,$$

where  $z$  is the number of detection opportunities. One detection opportunity is said to exist if

1. The sensor is monitored at the particular time (probability  $N_M/N$ ).
2. The sensor has been monitored for a time period  $T_m$ .
3. The submarine has been continuously snorkelling for that period, given it was snorkelling at the start of the period (probability  $\exp\{-T_m/T_s\}$ ).

$N_M$  is the number of simultaneously-monitored sensors and, as before,  $N$  is the total number of sensors in the field.

For a given total snorkelling distance  $\ell_s$ , there is maximum value for  $z$  of  $\text{Max}\{z\} = Q$ . Taking into account boundary effects, the mean value of  $Q$  is used:

$$\ell_s/V_s/T_m = 1;$$

$z$  is then binomially distributed:

$$\binom{Q}{z} p_m^z (1 - p_m)^{Q-z}, \text{ where } p_m = \frac{N_M}{N} \cdot \exp\{-T_m/T_s\}.$$

Since  $Q$  can take only integer values the lower integer value is chosen (Entier).

b) Deterministic Monitoring Schedule

Fixed groups of sensors are monitored sequentially, the next group to be monitored being that which has been left unmonitored the longest. We approximate  $p$  as

$$p = 1 - (1 - p_D)^{Q \cdot p_m},$$

where  $p_m$  has been found in Sect. 2.3.2a.

$E(p)$  and  $E(p^2)$  can then be found for the above two monitoring schedules for a given  $\ell_s$  (in the deterministic monitoring schedule we have  $E(p) = p$  and  $E(p^2) = p^2$ ). Thereafter new  $E(p)$  and  $E(p^2)$  can be found by averaging over  $\ell_s$  using the density function

$$f(\ell_s | y, T_s, T_d, U_s, U_d).$$

2.3.3 Submarine Path Relative to Sensor

The next stochastic variable to be treated is  $y$ , the path length of the submarine inside the detection range  $R$  of the sensor.

Besides the probability of choosing  $y$  as a constant, three distributions for  $y$  are used, representing increasing randomness of the submarine path. These are called the three sensor models, as described in the following sections. Before giving these three cases, a distribution is derived from which the first two distributions (S1 and S2) can be obtained as special cases. Consider Fig. 5.

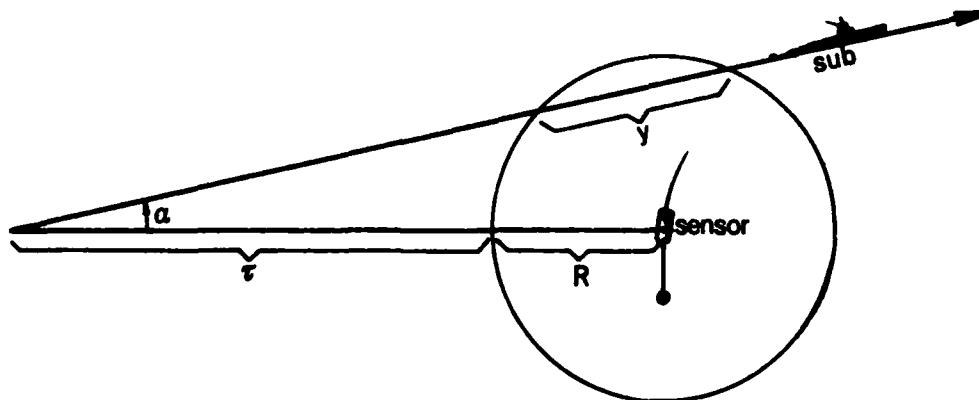


FIG. 5 DISTRIBUTION OF SUBMARINE PATH LENGTH

We seek the density function  $h$  of  $y$  with a uniform distribution of the angle  $\alpha$  between zero and  $\arcsin[(R/(\tau+R))]$ . The result is

$$h(y) = \frac{y}{2\sqrt{(2R)^2-y^2} \cdot \arcsin[(R/(\tau+R)) \cdot \sqrt{\tau^2+2\tau R+(y/2)^2}]}$$

a) Sensor Model S1

The submarine track is approximated with a straight line through the field. This model is therefore usable only for transiting submarines or patrolling submarines with straight-line paths comparable with the dimensions of the field. The straight-line path is reflected by letting  $\tau \rightarrow \infty$  in the expression for  $h(y)$  above. We find

$$h(y) = \frac{y}{2R \cdot \sqrt{(2R)^2 - y^2}}$$

In this model the closest point of approach (C.P.A.) between submarine and sensor is uniformly distributed between zero and  $R$ .

b) Sensor Model S2

The submarine track here does not have to be approximated with a straight line through the field, but is approximated with a straight line inside the detection range,  $R$ , of each sensor. This is reflected by setting  $\tau = 0$ . We find

$$h(y) = \frac{2}{\pi} \cdot \frac{1}{\sqrt{(2R)^2 - y^2}} .$$

c) Sensor Model S3

This last model is constructed by relaxing even the straight line approximation inside the detection range,  $R$ , of each sensor. Instead we assume a "Brownian motion" type of path inside the detection circle i.e. the point where the submarine leaves the detection circle is independent of where it entered. If  $P(y)$  denotes the probability of a submarine path length inside the detection circle greater than  $y$ , we have

$$P(y + \Delta y) = P(y) \cdot \left[ 1 - \frac{2R\Delta y}{\pi R^2} \right],$$

giving  $P(y) = \exp\{-2y/(\pi R)\}$  and therefore the density function

$$h(y) = \frac{2}{\pi R} \cdot e^{-\frac{2y}{\pi R}} .$$

This model is meant for the random patrol case where the straight-line pieces of the submarine path are small compared with  $R$  and where the alterations of course are significant.

As mentioned before, it is seen that the models S1, S2, and S3 represent increasing randomness of the submarine path. New  $E(p)$  and  $E(p^2)$  can now be found, taking into account the distribution  $h(y)$ , by averaging  $E(p)$  and  $E(p^2)$  over  $y$  with weight function  $h(y)$ , regarding the detection range  $R$  as a constant.

#### 2.3.4 Detection Range of Sensor

The detection range,  $R$ , can be regarded either as a constant or as having a truncated normal distribution. For further details, see Sect. 3.4.

### 3 COMPARISONS

#### 3.1 Distribution of Total Snorkelling Distance

With the analytical model the density function of the total snorkelling distance on a certain path length inside a sensor area (see Sect. 2.3.1) has been tested and has been compared with a straightforward simulation experiment.

The submarine's speed was assumed to be 8 kn while snorkelling and 4 kn while submerged. The lengths of the snorkelling and submerged periods were supposed to be negative exponentially distributed with a mean submerged time of 12 hours and a mean snorkelling time of 1 hour. These values have been retained throughout all the comparison runs with the simulation and analytical models.

In the simulation experiment a single sensor with a maximum contact range of 6 n.mi was considered as the sensor field. The conventional submarine was transiting exactly under the sensor, i.e. the path length inside the sensor area was always 12 n.mi. The state of the submarine when entering the maximum contact range of the sensor, and the lengths of snorkelling and submerged periods, were the three random variables in the experiment.

The probability that the submarine is snorkelling when it enters the 6 n.mi zone is

$$\frac{8}{48+8} = 1/7.$$

In each run a number is randomly drawn from the interval  $[0,1]$ ; if the number is smaller than  $1/7$  the submarine enters snorkelling, otherwise it is submerged. The methods of deciding the lengths of snorkelling and submerged periods have been described in Sect. 1.2.4. The total snorkelling distance inside the sensor area was measured for each run and Fig. 6 shows the density function derived from 2000 runs, together with the theoretical density function. Note that in more than 66% of the runs the submarine was not snorkelling at all inside the sensor area. The discrepancies between

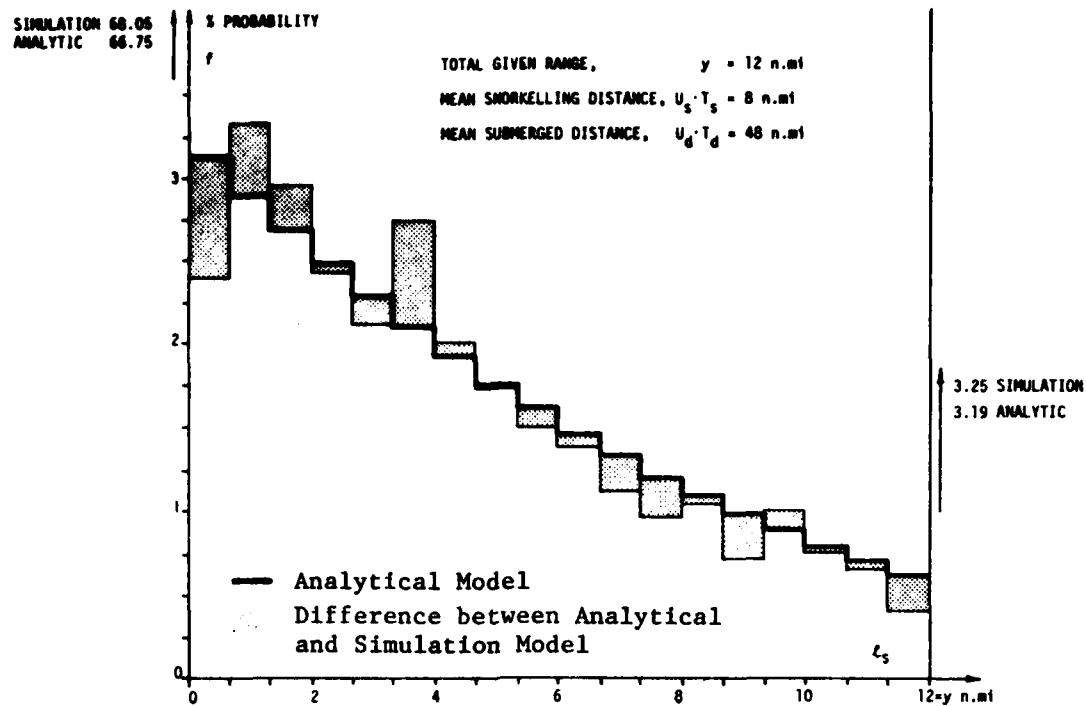


FIG. 6 DENSITY FUNCTION OF TOTAL SNORKELLING DISTANCE,  $l_s$

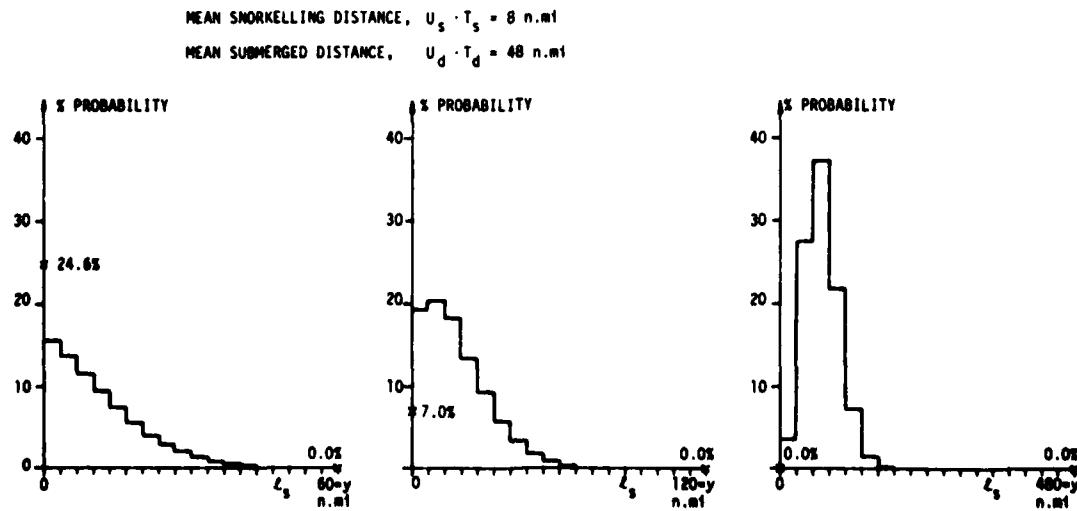


FIG. 7 DENSITY FUNCTIONS OF TOTAL SNORKELLING DISTANCE,  $l_s$ ,  
FOR GIVEN RANGE INTERVALS: 60, 120 AND 480 n.mi

the density functions can be caused by the number of runs in the simulation, which was too small to obtain a high accuracy in the results. For path lengths of 60, 120, and 480 n.mi the density functions calculated with the analytical model are given in Fig. 7.

### 3.2 Detection Probability per Sensor as Function of Submarine Path Length inside a Sensor Detection Area

A field of twelve sensors ( $N = 12$ ), of which eight are monitored simultaneously ( $N = 8$ ), has been studied. In the time-sharing schedule applied, a set of eight sensors is chosen randomly from the field of twelve sensors, and each set is monitored for five minutes ( $T_m = 5$  min).

A cookie-cutter detection model with a detection probability of  $p_D = 0.7$  has been used. In this experiment the cookie-cutter range of the sensor is not relevant, because the mean detection probability of one sensor will be projected on the path length of the submarine inside its cookie-cutter range.

The simulation has measured the detection performance of sensors with submarine path length of 4, 8, 12 and 16 n.mi. The configuration of the field used in this experiment is given in Fig. 8. The submarine is travelling a straight path exactly under the four sensors indicated.

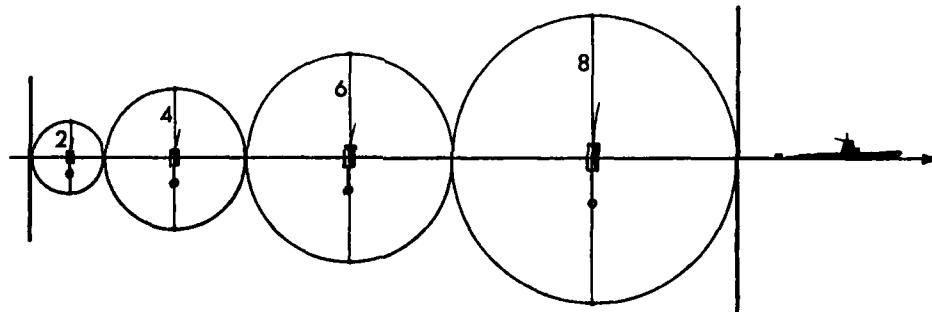
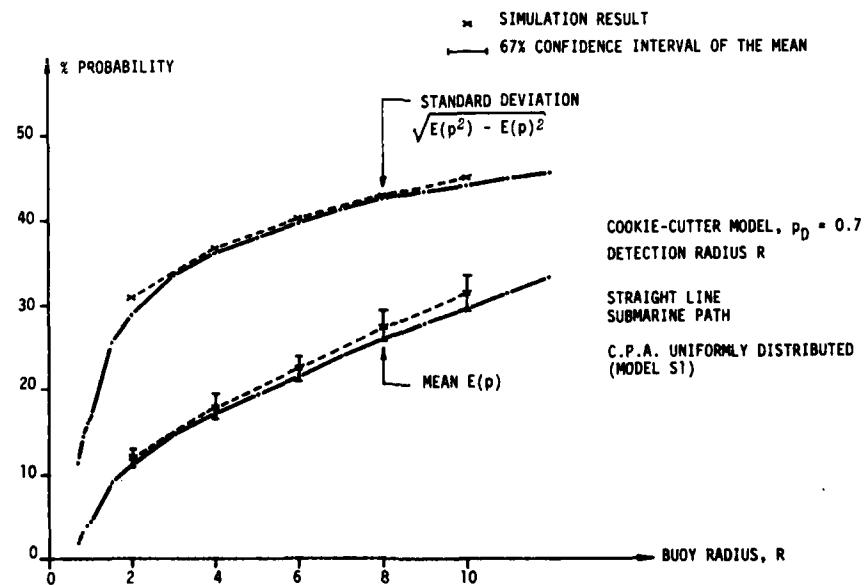
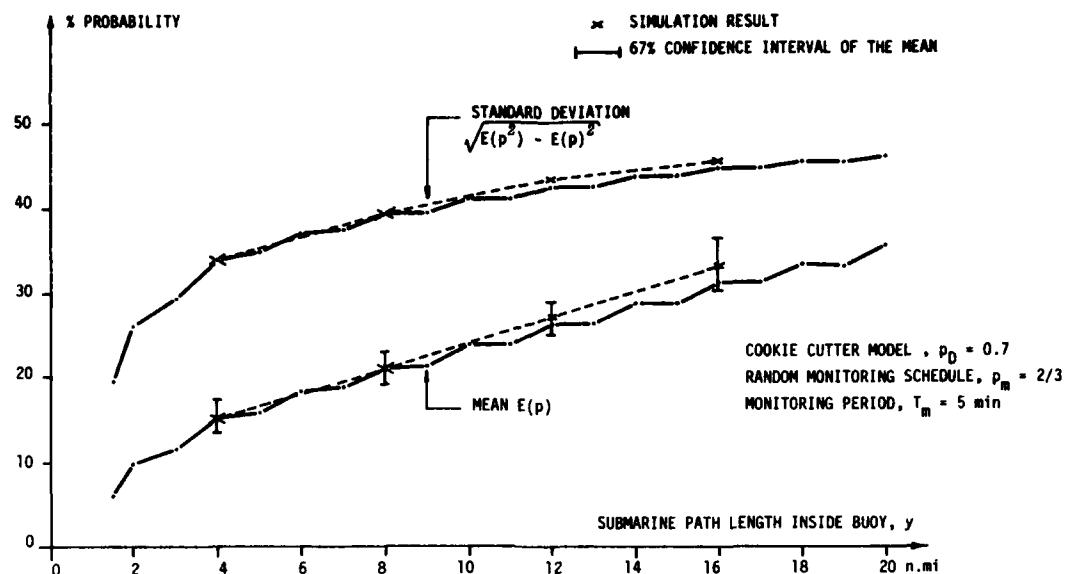


FIG. 8 CONFIGURATION OF THE FIELD USED IN THE EXPERIMENT

If a submarine snorkels continuously for 5 min inside a sensor detection circle and the sensor has been monitored during the same period, a detection opportunity occurs and the probability of detection is 0.7. If more than one detection opportunity takes place inside a sensor detection circle, these opportunities have been treated independently and the detection probability becomes  $1 - (1 - p_D)^z$ , where  $z$  is the number of detection opportunities.

The simulation deals with four random variables: the same three variables of the previous experiment and the random monitoring schedule.



The detection probability of each of the four sensors in Fig. 8 is measured for each run. After the  $i$ th run the mean detection probability  $\mu_i$  and standard deviation  $\sigma_i$  are calculated with the recursive formulas

$$\mu_i = \left(1 - \frac{1}{T}\right) \mu_{i-1} + \frac{1}{T} p_i ,$$

$$\sigma_i^2 = \left(1 - \frac{1}{T-1}\right) \sigma_{i-1}^2 + \frac{1}{T} (\mu_{i-1} - p_i)^2 ,$$

where  $\mu_{i-1}$  and  $\sigma_{i-1}$  are the mean and standard deviation calculated up to the  $(i-1)$ -th run, and  $p_i$  is the detection probability measured in the  $i$ th run.

The curves for mean detection probability and standard deviation presented in Fig. 9 were derived from Sect. 2.3.1 of the analytical model. The simulation results have also been plotted on this figure. According to the central limit theorem, the distribution of the mean values,  $\mu$ , tends to a normal distribution with standard deviation  $\sigma/\sqrt{n}$ , where  $n$  is the number of simulation runs. To indicate the error in the calculated mean, this standard deviation has also been noted on the figure.

### 3.3 Detection Probability per Sensor as a Function of the Sensor Detection Radius

This experiment used the same field configuration as in the previous experiment, except that the detection probability was projected on the cookie-cutter detection range.

The submarine was assumed to travel a straight path through the sensor detection circle with the closest point of approach uniformly distributed over the interval  $[0, R]$ , where  $R$  is the cookie-cutter detection range. The curves for mean detection probability and standard deviation calculated with the analytical sensor model S1 are presented in Fig. 10. In the simulation a fifth random variable has been introduced, i.e. the CPA of the submarine as shown in Fig. 11.

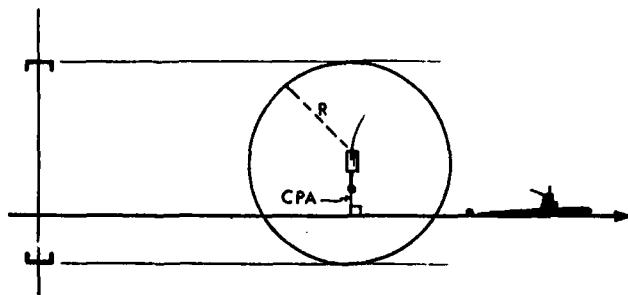


FIG. 11 CLOSEST POINT OF APPROACH TO SENSOR

The simulation model has been tested for  $R = 2, 4, 6, 8$  and  $10$  n.mi and the results shown in Fig. 10. For each detection range,  $R$ , 600 simulation runs were made. The standard deviation of the calculated mean has also been plotted in Fig. 10.

### 3.4 Detection Probability of a Sensor with a Smooth Detection Curve

The next step was to consider the detection range  $R$  not as a constant but as a random variable with a certain distribution, and thereby to relax the assumption about a cookie-cutter type of detection model.

The distribution of  $R$  was chosen to be a truncated normal distribution, as indicated in Fig. 12, in which the new detection model is shown as a dotted line. The truncated normal distribution has discrete probabilities at the ends.

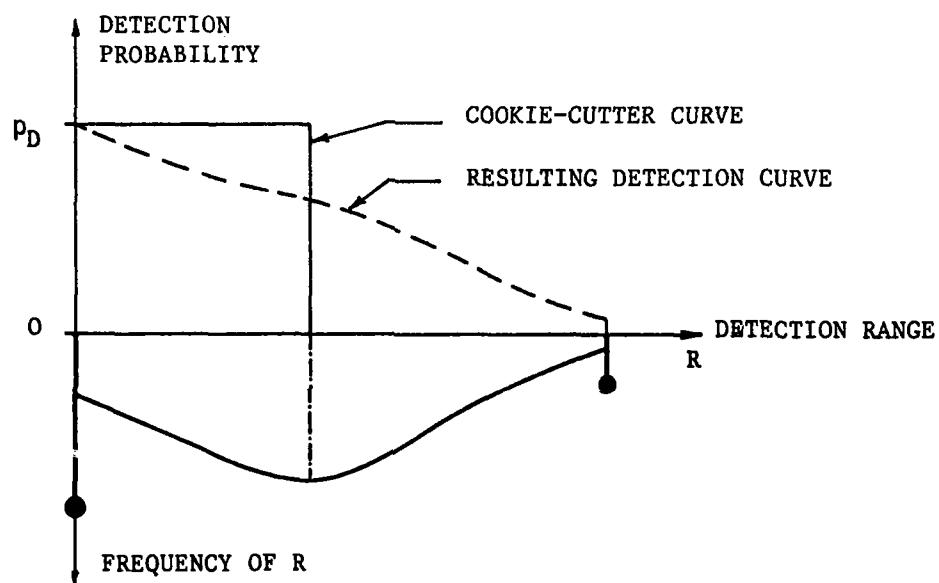


FIG. 12 DETECTION MODEL

The range of variation of  $R$  is chosen to be

$$\text{Max}\{0, R_m - 3 \cdot R_{sd}\} \leq R \leq R_m + 3 \cdot R_{sd}, \quad [\text{Ineq. 1}]$$

where  $R_m$  and  $R_{sd}$  are the mean and standard deviations of the untruncated normal distribution respectively.

In the analytical model the new and final  $E(p)$  and  $E(p^2)$  can be found by averaging  $E(p)$  and  $E(p^2)$  over  $R$ , using the above-mentioned truncated normal distribution as weight function.

A sixth random variable was introduced in the simulation: the values for the cookie-cutter range were drawn from a normal distribution with mean  $R_m = 3$  n.mi and standard deviation  $R_{sd} = 3$  n.mi. At each run one sample value for  $R$  was drawn, but if the value of  $R$  lays outside one of the limits of Ineq. 1 then the limit value for  $R$  was taken. Thereafter, as in the previous experiment, the CPA value was drawn randomly from the interval  $[0, R]$ . The results of both models are shown in Table 1. 1000 runs were made with the simulation model, which gave a standard deviation of the mean of 0.01.

TABLE 1  
DETECTION PROBABILITY PER SENSOR

	Analytical Model	Simulation
Mean $E(p)$	0.13	0.14
Standard deviation $\sqrt{E(p^2) - E(p)^2}$	0.32	0.33

The analytical model needed one CPU (Central Processing Unit) minute for the calculation (Table 1) and the simulation model needed 42 CPU minutes for the 1000 runs.

### 3.5 Field Diagram

For a given type of field and a given submarine behaviour, a diagram is produced (called "field diagram") on which can be read the mean detection probability ( $p_M$ ) for a given total number ( $N$ ) of sensors in the field and a given number of sensors monitored at the same time ( $N_M$ ). The following scenario was chosen:

The field: A varying number of sensors ( $N$ ) are placed in four ( $M = 4$ ) parallel lines with equal spacing between sensors. The lines are 100 n.mi long and are displaced randomly relative to each other (i.e. in the analytical model field model F1 applies). A random monitoring schedule is chosen, with  $N_M$  sensors monitoring simultaneously.

The sensor: A cookie-cutter detection model is taken, with a detection range  $R = 3$  n.mi and  $p_D = 0.7$ . Since field model F1 does not accept overlap of the sensor detection circles, the condition  $N \leq 66$  holds.

The submarine: The same assumptions as in the previous experiments are made for the snorkelling policy. The track of the submarine is chosen to be a straight line perpendicular to the lines of sensors (i.e. in the analytical model, sensor model S1 applies).

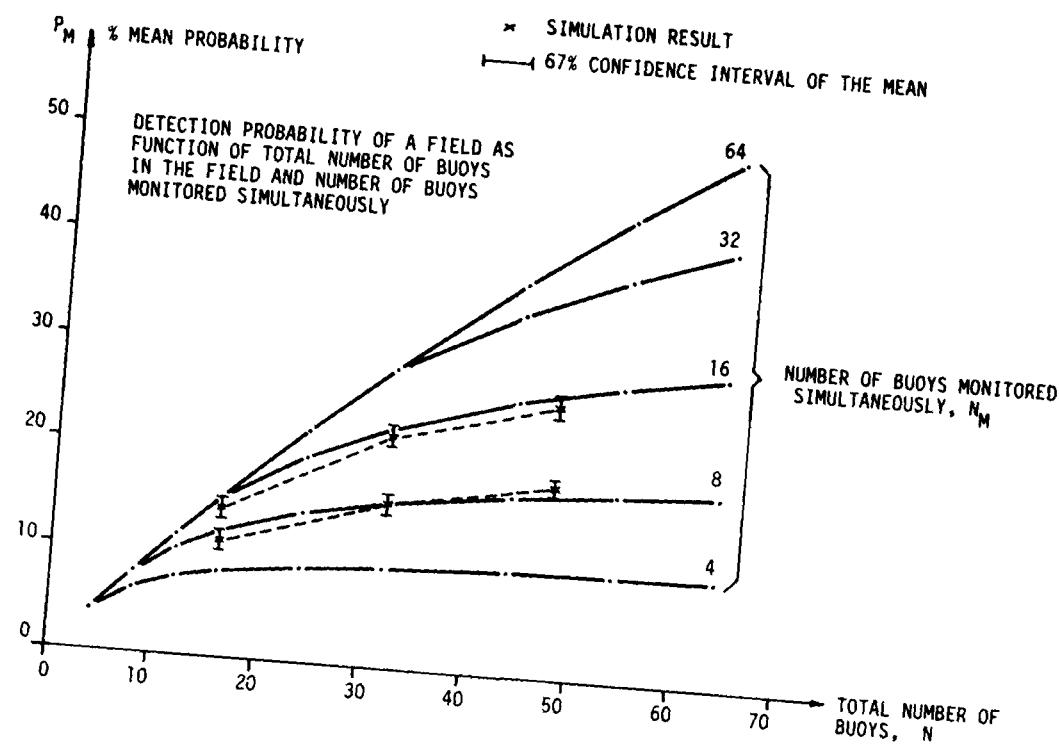


FIG. 13 FIELD DIAGRAM

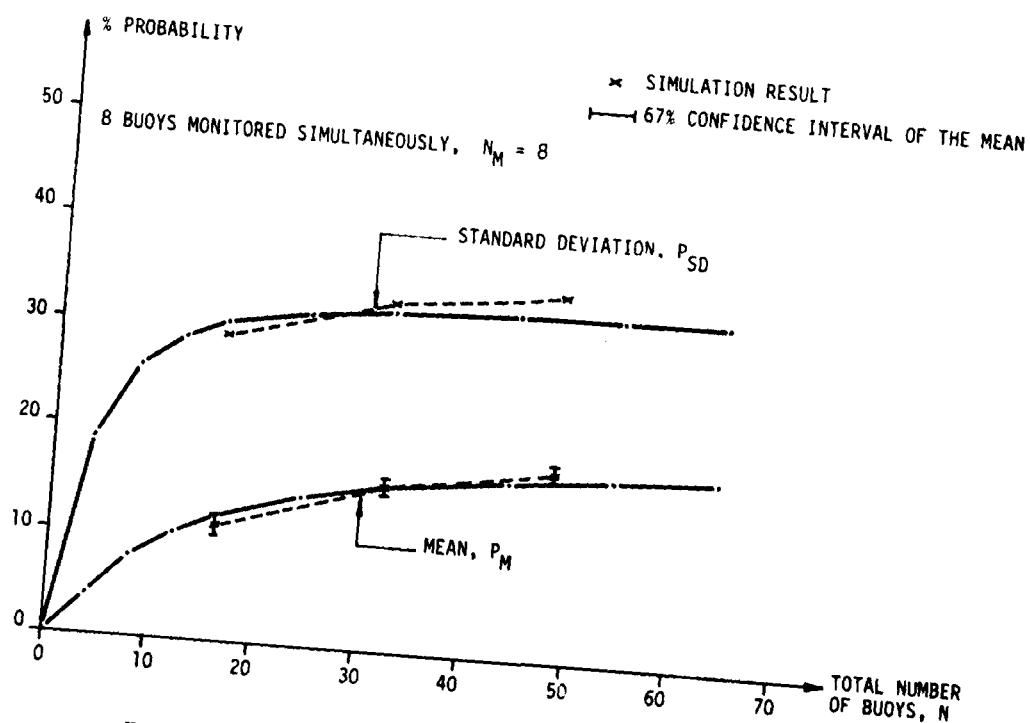


FIG. 14 MEAN AND STANDARD DEVIATION OF THE DETECTION PROBABILITY OF A FIELD

The simulation model dealt with six random variables: the lengths of the submerged and snorkelling periods (negative exponentially distributed), the state of the submarine when entering the field area, the entry point of the submarine into the field, the monitoring schedule, and the relative displacement of the lines of sensors.

The field diagram is shown in Fig. 13. Simulation runs have been made for fields with  $N = 16, 32, 48$  and  $N_M = 8$  and  $16$ . The (from 800 runs) calculated mean probabilities of detection and the standard deviations of the means are also presented in the field diagram. Moreover, one of the curves ( $N_M = 8$ ) of the field diagram has been removed to show also the course of the standard deviation of the distribution of the detection probabilities (see Fig. 14).

### CONCLUSIONS

The two models described have been tested for a field of long-life sensors monitored by MPA (Maritime Patrol Aircraft) in the southwestern entrance of the English Channel. Since the differences in the results are small it is concluded that both models work satisfactorily in this experiment.

Both models could be applied very well for comparing detection performances of different passive systems. Although the simulation model has more options it consumes considerably more computer time than the analytical model.

Referring to the aim of this study, obviously the effectiveness of a distributed field of passive sensors cannot only be expressed in its detection performance. Such parameters as accuracy of datum, time-late of searcher after a detection, tracking possibilities, etc. are characteristics of a passive system that have a main influence on the localization and attack performance. Therefore the simulation model should be extended to localization and attack phases.

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APPENDIX ADENSITY FUNCTION OF THE TOTAL TIME FOR  
TWO STRICTLY ALTERNATING POISSON PROCESSES

For the two alternating Poisson processes described in Sect. 2.3.1 we find the density function

$$g(t_s | t^*, T_s, T_d) = \sum_{n=0}^{\infty} \frac{\left(\frac{t^* - t_s}{T_d}\right)^n}{n!} e^{-\frac{t^* - t_s}{T_d}} \cdot f_{s,n}(t_s) +$$

$$\sum_{m=0}^{\infty} \frac{\left(\frac{t_s}{T_s}\right)^m}{m!} e^{-\frac{t_s}{T_s}} \cdot f_{d,m+1}(t^* - t_s),$$

where  $f_{s,n}(t_s)$  and  $f_{d,m+1}(t^* - t_s)$  are the density functions (nth and  $m+1$ th convolutions) of the total snorkelling ( $t_s$ ) and submerged ( $t^* - t_s$ ) time for a given number of single snorkelling ( $n$ ) and submerged ( $m+1$ ) periods respectively, given that the submarine is in the submerged state at the start of the time interval  $t^*$ .

Looking at the first term of  $g(t_s | t^*, T_s, T_d)$

$$H(t_s) = \sum_{n=0}^{\infty} \frac{\left(\frac{t^* - t_s}{T_d}\right)^n}{n!} e^{-\frac{t^* - t_s}{T_d}} \cdot f_{s,n}(t_s),$$

we make use of the Laplace transform from the  $t_s$  domain to the S domain. We find

$$L\{H(t_s)\} = e^{-\frac{t^*}{T_d}} \left[ 1 + \sum_{n=1}^{\infty} \frac{t^{*n}}{(n-1)!n!(T_s T_d)^n} \sum_{i=0}^n \left(-\frac{1}{t^*}\right)^i \binom{n}{i} (n+i-1)!$$

$$\left( \frac{1}{S + \frac{1}{T_s} - \frac{1}{T_d}} \right)^{n+1} \right]$$

Going back to the  $t_s$  domain at this stage we obtain

$$H(t_s) = e^{-\frac{t^*}{T_d}} \left[ \delta(t_s) + \frac{1}{t_s} e^{-(\frac{1}{T_s} - \frac{1}{T_d})t_s} \sum_{n=1}^{\infty} \frac{\left( \frac{t_s(t^*-t_s)}{T_s T_d} \right)^n}{(n-1)! n!} \right],$$

where

$$\sum_{n=1}^{\infty} \frac{\left( \frac{t_s(t^*-t_s)}{T_s T_d} \right)^n}{(n-1)! n!} = \sqrt{\frac{t_s(t^*-t_s)}{T_s T_d}} \cdot I_1\left(2\sqrt{\frac{t_s(t^*-t_s)}{T_s T_d}}\right)$$

and  $\delta$  and  $I_1$  are the dirac function and the modified Bessel function of the first kind and first order respectively.

Treating the second term for  $g(t_s|t^*, T_s, T_d)$  above in the same way we obtain the expression for  $g(t_s|t^*, T_s, T_d)$  given in the main text.